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## Tensor Polarizabilities of Magnetoelectric Particles on the Base of Strip-Line-Coupled Magnetostatic Wave Resonators

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#### Abstract

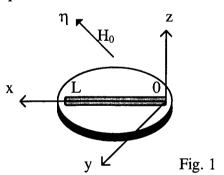
An electrodynamic description of bianisotropic particles on the base of strip-line-coupled magnetostatic wave (MSW) resonators is developed and analytical closed-form expressions for their tensor polarizabilities are obtained for an arbitrary direction of a magnetizing field and an arbitrary resonator shape. Numerical calculations are performed for a normally magnetized thin-ferrite-film disk resonator with a metal strip on its surface.

#### 1. Introduction

Recently proposed composite non-reciprocal bianisotropic materials [1] represent ensembles of magnetized thin-ferrite-film magnetostatic wave resonators with surface metallization. Induced electric and magnetic dipole moments  $\mathbf{p}_e$  and  $\mathbf{p}_m$  of such artificial particles are related to the external electric and magnetic fields as

$$\mathbf{p}_{e} = \ddot{\alpha}_{ee} \mathbf{E} + \ddot{\alpha}_{em} \mathbf{H} , \quad \mathbf{p}_{m} = \ddot{\alpha}_{me} \mathbf{E} + \ddot{\alpha}_{mm} \mathbf{H} , \qquad (1)$$

where  $\ddot{\alpha}_{ee}$ ,  $\ddot{\alpha}_{em}$ ,  $\ddot{\alpha}_{me}$  and  $\ddot{\alpha}_{mm}$  are corresponding tensor polarizabilities. Although a nature and qualitative physical picture of magnetoelectric coupling in these elements is quite obvious and an experimental evidence of the effect has been obtained [2], the electrodynamic description of the



particles and quantitative evaluation of magnetoelectric coupling till the present time has not been performed. In this paper we consider a thin-ferrite-film resonator with a linear metal strip of the width b on its surface, as shown in the Fig.1 for a disk resonator magnetized by the field  $H_0$  in an arbitrary direction  $\eta$ . Dimensions of the resonator are much less than a wavelength in the surrounding media with the permittivity  $\varepsilon$ , so electric and magnetic fields in (1) can be assumed uniform and quasistatic. Magnetization of oscillation modes  $\mathbf{m}_q$  for the type q and corresponding eigenfrequencies  $\omega_q$  are supposed to be known, as well the ferromagnetic resonance linewidths

 $\Delta H_q$ . The approach is based on the earlier obtained solutions of self-consistent electrodynamic problems of the excitation of one-port [3] and two-port [4] MSW resonators. In this formulation, in particular, dipole moments of a particle, induced by an external magnetic field are found through a resonator high-frequency magnetization, which is determined taking into account the "back" influence of the current in the strip on this magnetization. Neglecting of this interaction results in the non-accurate determination of the polarizabilities  $\ddot{\alpha}_{em}$ ,  $\ddot{\alpha}_{mm}$  and their resonant frequency. Presented electrodynamic description of magnetoelectric particles enables to obtain constitutive relations for composite media in a closed form.

# 2. Self-Consistent Electrodynamic Problem for $\ddot{\alpha}_{\it em}$ and $\ddot{\alpha}_{\it mm}$ Determination

Tensor polarizabilities  $\ddot{\alpha}_{em}$  and  $\ddot{\alpha}_{mm}$  are calculated in the assumption that  $\mathbf{E}=0$  in (1). Magnetization induced by a given magnetic field  $\mathbf{H}$  is found as an expansion into the series of eigenmodes

$$\mathbf{M} = \sum_{q} c_{q} \mathbf{m}_{q} , \quad c_{q} = \varphi_{q} \int_{V} (\mathbf{H} + \mathbf{h}_{\mu}) \cdot \mathbf{m}_{q}^{*} dV , \qquad (2)$$

where

$$\varphi_q = -\frac{b_q}{\omega^2 - \omega_q^2 - i\omega_q^2 Q_q^{-1}}, \qquad b_q = \frac{i(\omega + \omega_q)\omega_M}{\Phi_q}, \qquad \Phi_q = \int_V \left[\mathbf{m}_q \times \mathbf{m}_q^*\right] dV,$$

$$Q_q = \frac{H_0}{2\Delta H_q}$$
,  $\omega_M = \gamma \mu_0 M_0$ ,  $\gamma$  is the gyromagnetic ratio,  $\mu_0$  is the permeability of free space,  $M_0$ 

is the saturation magnetization, and V is the resonator volume. The "back" influence of a strip current on the magnetization is taken into account by including the magnetic field  $\mathbf{h}_{\mu}$  in (2), which can be found as a field of a transmission line excited by an equivalent given magnetic current  $i\omega \mathbf{M}$ 

$$\mathbf{h}_{\mu} = c_{\mu}(x)\mathbf{H}_{\mu}^{r} + c_{-\mu}(x)\mathbf{H}_{-\mu}^{r} , \qquad (3)$$

with

$$c_{\mu}(x) = -\frac{i\omega\omega_0}{N_{\mu}^r} \int_0^x dx \int_S (\mathbf{M} \cdot \mathbf{H}_{-\mu}^r) dS, \quad c_{-\mu}(x) = -\frac{i\omega\omega_0}{N_{\mu}^r} \int_x^L dx \int_S (\mathbf{M} \cdot \mathbf{H}_{-\mu}^r) dS$$

where

$$\mathbf{H}_{\mu}^{r} = \mathbf{H}_{\mu} + \Gamma_{2} \mathbf{H}_{-\mu}$$
,  $\mathbf{H}_{-\mu}^{r} = \mathbf{H}_{-\mu} + \Gamma_{1} \mathbf{H}_{\mu}$ ,  $N_{\mu}^{r} = (1 - \Gamma_{1} \Gamma_{2}) N_{\mu}$ ,

 $\mathbf{H}_{\pm\mu} = \mathbf{H}_{\pm\mu0}(y,z)exp(\mp i\beta x)$  is the magnetic field of a dominant wave in a strip-line (in the absence of ferrite) with the propagation constant  $\beta$ ,  $N_{\mu}$  is a normalization coefficient [4],  $\Gamma_1$  and  $\Gamma_2$  are the reflection coefficients for a strip current at x=0 and x=L ( $\Gamma_1=-1$  and  $\Gamma_2=-exp(-2i\beta L)$  for our case of an open transmission line), and S is the cross section of the line. The system of integral equations (2), (3), formulating a self-consistent electrodynamic problem, has the following solution for amplitudes of electromagnetic waves in the transmission line

$$c_{\mu}(x) = \left[ \left( e^{i\beta x} - 1 \right) - \Gamma_{1} \left( e^{-i\beta x} - 1 \right) \right] c_{\mu 0}, \quad c_{-\mu}(x) = \left[ \left( e^{-i\beta x} - e^{-i\beta L} \right) - \Gamma_{1} \left( e^{i\beta x} - e^{i\beta L} \right) \right] c_{\mu 0}, \quad (4)$$

where 
$$c_{\mu 0} = -\frac{i\omega\mu_0}{N_\mu} \left[ i\beta(1-\Gamma_1\Gamma_2) + 2s\frac{i\omega\mu_0}{N_\mu} \phi_q |I_{\mu q}|^2 \right]^{-1} I_{\mu q}^* \phi_q \int_V (\mathbf{H} \cdot \mathbf{m}_q^*) dV$$
,  $I_{\mu q} = \int_S (\mathbf{H}_{\mu 0} \cdot \mathbf{m}_q^*) dS$ 

and parameter s, depending on the line length, is given in [4] ( $s \approx 2i\beta L^2$  for a special case  $\beta L \ll 1$ ). Substituting (4) in (3) and using Ampere's law (an integration extends over an arbitrary contour C in the cross section yz which includes the strip)

$$\oint_C \mathbf{h}_{\mu} d \mathbf{l} = J(x),$$

one can find an induced electric dipole moment and using (2) - a magnetic dipole moment

$$\mathbf{p}_{e} = \frac{1}{i\omega} \mathbf{e}_{x} \int_{0}^{L} J(x) dx, \ \mathbf{p}_{m} = \sum_{q} c_{q} \int_{V} \mathbf{m}_{q} dV.$$
 (5)

Polarizabilities can be written in a tensor form

$$\ddot{\alpha}_{em} = A \cdot \begin{bmatrix} m_{qx}^* & m_{qy}^* & m_{qz}^* \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \ \ddot{\alpha}_{mm} = B \cdot \begin{bmatrix} m_{qx} m_{qx}^* & m_{qx} m_{qy}^* & m_{qx} m_{qz}^* \\ m_{yx} m_{qx}^* & m_{qy} m_{qy}^* & m_{qy} m_{qz}^* \\ m_{qz} m_{qx}^* & m_{qz} m_{qy}^* & m_{qz} m_{qz}^* \end{bmatrix},$$
 (6)

where for the case  $\beta L \ll 1$  coefficients A and B are the following

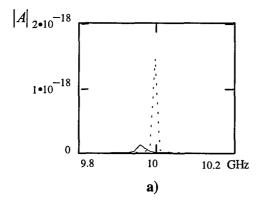
$$A = \frac{i}{6} \frac{\mu_0}{\sqrt{2WN_{\mu}}} \varphi_q I_{\mu q}^* V \beta^2 L^3 \left( -\beta + L \frac{\omega \mu_0}{N_{\mu}} \varphi_q \left| I_{\mu q} \right|^2 \right)^{-1}, \tag{7}$$

$$B = \varphi_q V^2 + \varphi_q^2 V^2 \left| I_{\mu q} \right|^2 \frac{\omega \mu_0 \beta^3 L^4}{3N_{\mu}} \left[ i\beta (1 - \Gamma_1 \Gamma_2) + 2s \varphi_q \left| I_{\mu q} \right|^2 \frac{i\omega \mu_0}{N_{\mu}} \right]^{-1}, \tag{8}$$

and W is a characteristic impedance of a transmission line, which can be calculated as [5]

$$W = \frac{120}{\sqrt{\varepsilon}} \left( \ln \frac{2,2L}{b} - 1 \right). \tag{9}$$

For a normally magnetized ferrite disk and a fundamental uniform mode of magnetization (when its components can be assumed to be  $m_{\rm qx}=1$ ,  $m_{\rm qy}=-i$ ,  $m_{\rm qz}=0$ ) calculated polarizability coefficients A (7) are shown in the Fig.2. Note that in this case an internal biasing magnetic field is uniform. The diameter of the resonator is taken equal to the length of a strip and the particle is characterized by the following set of parameters: L=0.5mm,  $b=20\mu m$ ,  $4\pi M_0=1750$  Oe,  $H_0=5320$  Oe,  $2\Delta H_0=0.5$  Oe,  $\epsilon=10$ .



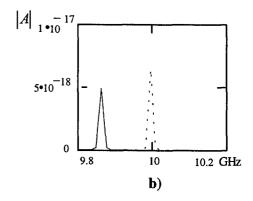


Fig. 2 Polarizability coefficient |A| (ferrite film thickness: a) 10  $\mu$ m, b) 50  $\mu$ m). Solid curve: Self-consistent solution; Dotted: "Back" coupling neglected.

### 3. Calculation of Tensor Polarizabilities $\ddot{\alpha}_{\it ee}$ and $\ddot{\alpha}_{\it me}$

Tensor polarizabilities  $\ddot{\alpha}_{ee}$  and  $\ddot{\alpha}_{me}$  are calculated in the assumption that  $\mathbf{H} = 0$  in (1). A current distribution along a narrow strip in an external longitudinal electric field  $\mathbf{E}$  is known from the antenna theory [5]

$$J(x) = J(L/2)f(x), \tag{10}$$

where 
$$J(L/2) = \frac{2iE}{\beta W \cos(\beta L/2)} \left(1 - \cos\frac{\beta L}{2}\right)$$
,  $f(x) = \frac{\cos\beta(x - L/2) - \cos(\beta L/2)}{1 - \cos(\beta L/2)}$ . Induced electric

dipole moment  $\mathbf{p}_{e}$  is obtained by integrating the current J(x) according (5). A magnetic dipole moment

 $\mathbf{p}_{\rm m}$  of the ferrite resonator is excited by the current (10) and can be calculated using (5) with the amplitude coefficients  $c_{\rm q}$  being obtained from (2) for  $\mathbf{H}=0$ . It can be easily seen that the magnetic field of the transmission line is related to the current as

$$\mathbf{h}_{\mu} = \frac{J(x)}{J_c} \mathbf{H}_{\mu 0} \,, \tag{11}$$

where  $J_c = \oint_C \mathbf{H}_{\mu 0} d\mathbf{l}$  (for this value the following relationship is valid  $J_c = \sqrt{N_{\mu}/(2W)}$ ).

Performing calculations we come to the following polarizabilities for the case  $\beta L <<1$ 

$$\ddot{\alpha}_{ee} = C \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \ddot{\alpha}_{me} = D \cdot \begin{bmatrix} m_{qx} & 0 & 0 \\ m_{qy} & 0 & 0 \\ m_{qz} & 0 & 0 \end{bmatrix}, \tag{12}$$

where 
$$C = \frac{1}{6} \frac{\beta L^3}{\omega W}$$
,  $D = \frac{i\beta L^3}{6W} \frac{\varphi_q I_{\mu q} V}{J_c}$ .

#### 4. Conclusion

Derived explicit expressions for polarizabilities show that all non-zero tensor elements, except  $\ddot{\alpha}_{ee}$ , have resonant character with resonance frequencies close, but not equal, to the MSW resonator eigenfrequencies. Elements of tensors of magnetoelectric coupling  $\ddot{\alpha}_{em}$ ,  $\ddot{\alpha}_{me}$  are proportional to the saturation magnetization  $M_0$  of ferrite and to the third power of the strip length  $L^3$  and increase with the growth of the resonator quality factor Q. For low magnetic losses these tensors are related as  $\ddot{a}_{em} \approx \mu_0 \ddot{a}_{me}^{T*}$ . Elements of the tensor  $\ddot{\alpha}_{mm}$  are proportional to the resonator volume V. The tensor  $\ddot{\alpha}_{ee}$  has no frequency dependence and is proportional to the third power of the strip length  $L^3$ . In the case when a ferrite resonator has a non-elliptic form (e.g., a straight-edge MSW resonator with a film of a rectangular form), calculations of polarizabilities must be carried out taking into account the influence of nonuniformity of the internal biasing magnetic field [6].

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